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Reviews

Edited by Tom Archibald and Scott Walter

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Most reviews are solicited. However, colleagues wishing to review a book are invited to make their wishes known to the appropriate Book Review Editor. (Requests to review books written in the English language should be sent to Tom Archibald at the above address; requests to review books written in other languages should be sent to Scott Walter at the above address.) We also welcome retrospective reviews of older books. Colleagues interested in writing such reviews should consult first with the appropriate Book Review Editor (as indicated above, according to the language in which the book is written) to avoid duplication.

The Mathematical Papers of Sir William Rowan Hamilton

Edited by Brendan K.P. Scaife. Cambridge (Cambridge University Press). 2000. pp. viii+842.

William Rowan Hamilton (1805–1865) is an attractive figure to historians of science. In contrast to the typical 19th-century mathematician, his life and work have been studied from the broader perspective of intellectual history. Although he made major contribution to theoretical physics and algebra, there is more to be learned from his career than just his mathematical science, a fact that is clear in Thomas Hankins's biography *Sir William Rowan Hamilton* (1980). Living in Dublin at the geographical periphery of British science, he was educated at a time of ferment and reform in the mathematical and scientific curriculum of British universities. His works were influenced by Immanuel Kant's philosophy, while he loved literature and had extensive contacts (primarily of an epistolary sort) with Francis Beaufort Edgeworth, William Wordsworth, and Samuel Taylor Coleridge, leading representatives of Romanticism in early 19th-century England.

In 1925 Arthur William Conway and John Lighton Synge began a project to produce an edited edition of Hamilton's collected papers. There followed volume one, "Geometrical Optics," ed. Conway and Synge in 1931; volume two, "Dynamics," ed. Conway and A.J. McConnell in 1940; and volume three,

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"Algebra," ed. H. Halberstam and R.E. Ingram in 1967. The book under review, edited by Brendan K.P. Scaife, completes the project. Its more diverse contents are indicated by the subtitle: Geometry, Analysis, Astronomy, Probability and Finite Difference and Miscellaneous. It contains Hamilton's published papers related to these subjects as well as three previously unpublished manuscripts: the third part of the "Systems of Rays," two letters to Augustus De Morgan and a very long letter and postscript addressed to Andrew Searle Hart on anharmonic coordinates. Among the 55 papers listed in the table of contents, approximately one-fourth are given only by their title and place of publication.

In place of the appendices that had appeared in earlier volumes, the editor of volume four simply inserts footnotes in the body of each paper. The volume contains a list of Hamilton's work in approximate chronological order, a combined index for all four volumes, and a CD-Rom that includes the four volumes as PDF electronic files.

Hamilton's research is conventionally grouped in the subject areas of geometrical optics, analytical dynamics, and the study of new properties of negative and complex numbers and quaternions. Even without volume four we may feel that we have sufficient published material for an historical discussion of his work: the three volumes of his collected works and his books *Lectures on Quaternions* (Dublin, 1853) and *Elements of Quaternions* (London, 1866). In addition, Robert P. Graves' three-volume biography of 1882–1891 brought together several of his poems and extensive selections from his correspondence. Nevertheless, the fourth volume provides much new information concerning Hamilton's accomplishments, and is a welcome addition to Hamiltonian historiography.

The fourth volume devotes about 260 pages to Hamilton's work on "Anharmonic Coordinates," presented in his long letter to Hart (1811–1890), a senior fellow and bursar of Trinity College, Dublin. Since only a short account was published in 1860, the letter is a crucial source for understanding Hamilton's ideas on this subject. The letter consists of 490 sections and was written from February 27, 1860 to November 24, 1860. Together with several of his mathematical contemporaries, Hamilton was very interested in projective and line geometries, and cited the work of August Ferdinand Möbius, Michel Chasles, Arthur Cayley, Otto Hesse, and Joseph Serret. Hamilton's development of anharmonic coordinates possessed a very original character, as there were close relations between his ideas in this subject and his work on optics, quaternions, symbolic algebra, and hodographs.

The concept of anharmonic coordinates is not current today, and was connected to Hamilton's own particular development of projective geometry. Anharmonic coordinates or projective point coordinates are a kind of a projective analogue of Cartesian coordinates. Consider a triangle OXY and a point U in the plane of the triangle. Define the projections of U on OX and OY as the intersections A and B respectively of the lines OX with YU and OY with XU. OX, OY, and U correspond respectively to the x-axis, the y-axis, and the point $(1, 1)$ in Cartesian coordinates. Given an arbitrary point P in the plane, let Q and R be its projections on the lines OX and OY. The anharmonic coordinates x, y, z of the point P are defined as $\frac{x}{z} = \frac{OQ}{QX} \cdot \frac{XA}{AO}$, $\frac{y}{z} = \frac{OR}{RY} \cdot \frac{YB}{BO}$. Although Hamilton introduced these coordinates without the explicit notion of projective plane, they evidently pertain to what we would today call projective geometry. Hamilton also used the term anharmonic when he discussed several notions related to his development of geometry: the anharmonic ratio (better known today as the cross ratio), anharmonic coordinates of the line (known today as projective line coordinates), anharmonic quaternions, and the anharmonic of a pencil of lines. Following Cayley's precedent, he used notation for determinants in his writings. To trace Hamilton's theoretical development of this subject will be to enter an unexplored part of the history of geometry. The editorial notes to volume four provide little assistance in understanding this difficult subject.

There are several writings produced here that will help the reader greatly in evaluating Hamilton's achievements in optics and analysis. His major work of geometrical optics consists of the three parts of "Systems of Rays" and its three supplements. Volume one contained most of this material, although only the index was included for the third part of "Systems of Rays." The publication now of the third part will help us to follow those developments in geometrical optics that led him to the prediction of conical refraction and to dynamical theory.

In two letters to De Morgan of 1858, Hamilton discussed definite integrals and divergent series (February 15th) and the solution of third-order differential equations (July 15th). He arrived at these results through the long and diverse use of differential and integral calculus in his researches. Since the relevant parts of these subjects were well established on the Continent, an examination of Hamilton's results should help us to evaluate his achievements in comparison with those of continental mathematicians.

In addition to the easy access to Hamilton's important mathematical works afforded here, readers will enjoy discovering Hamilton's original thoughts in various places in his short essays and letters: the idea of the sympathy between poetry and science, the relation of arithmetic, metrology, and algebra, the nature of analysis and synthesis, and so on. The letters he wrote in his last years reveal his goal of linking mathematics closely with philosophy. Although these shorter essays contain few new scientific insights, they offer important clues concerning what he sought in his work.

Hankins's biography of 1980 has become an important source for the study and analysis of Hamilton's science. In addition, over the past several decades historians of mathematics have composed detailed specialized studies of his work in dynamics and algebra. The volume under review presents previously unexamined and important materials that shine fresh light on Hamilton and will enable us to read his papers from new viewpoints.

Michiyo Nakane
Department of Economics, Seijo University, Seijo, Setagaya, Tokyo, 157-8511 Japan
E-mail address: michiyo.nakane@nifty.com

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From Kant to Hilbert: A Source Book in the Foundations of Mathematics
Selected and edited by William Ewald. Oxford (Clarendon Press). 1996. Two volumes. xviii + 1340 pp. with a personal name index. $300 US

According to Joseph Dauben's authoritative bibliography *The History of Mathematics from Antiquity to the Present*, source books are relatively recent in mathematics. The earliest mentioned are by Andreas Speiser (1925) and David E. Smith (1929). Among source books treating only part of mathematics, an eminent example is Jean van Heijenoort's *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* (1967), which has been well regarded by logicians for decades. Indeed, it served as a model for the source book under review, whose title echoes "From Frege to Gödel" with "From Kant to Hilbert."

Van Heijenoort's source book served Ewald as the origin not merely of his title but also of its basic format. Happily for the reader, Ewald (like van Heijenoort) introduces each selection by describing its